Engset formula:

We considered a system with C channels and U users in Erlang formulation, where U >> C. We defined

$A_u = H\lambda_1$ erlangs, where λ_1 is average call arrival rate for one user, and

$A = UA_u = H\lambda$:- $\lambda = U\lambda_1$, the average call arrival rate in the system.

In micro-cellular networks, U >> C assumption does not hold, since the trunk contains small number of channels to serve a limited number of users.

The probability of having a call arrival in $[k\delta, (k+1)\delta]$ in this case, when there are no users in the system (N_k=0) is

$$\Pi_{01}[k+1, k] = U(\lambda_1\delta)\exp(-\lambda_1\delta) \approx U\lambda_1\delta$$

Note that this expression is exactly the same as the expression for $\Pi_{01}[k+1, k]$ in Erlang formulation, with λ replaced by $U\lambda_1$.

When the state N_k=1, number of users who can make a new call is U-1. Hence, the probability of having a call arrival in $[k\delta, (k+1)\delta]$, when there is one user in the system (N_k=1) is

$$\Pi_{12}[k+1, k] \approx (U-1)\lambda_1\delta.$$

Similarly, when there are i users in the system, the number of users that can make a new call is U-i, and hence

$$\Pi_{i,i+1}[k+1, k] \approx (U-i)\lambda_1\delta.$$

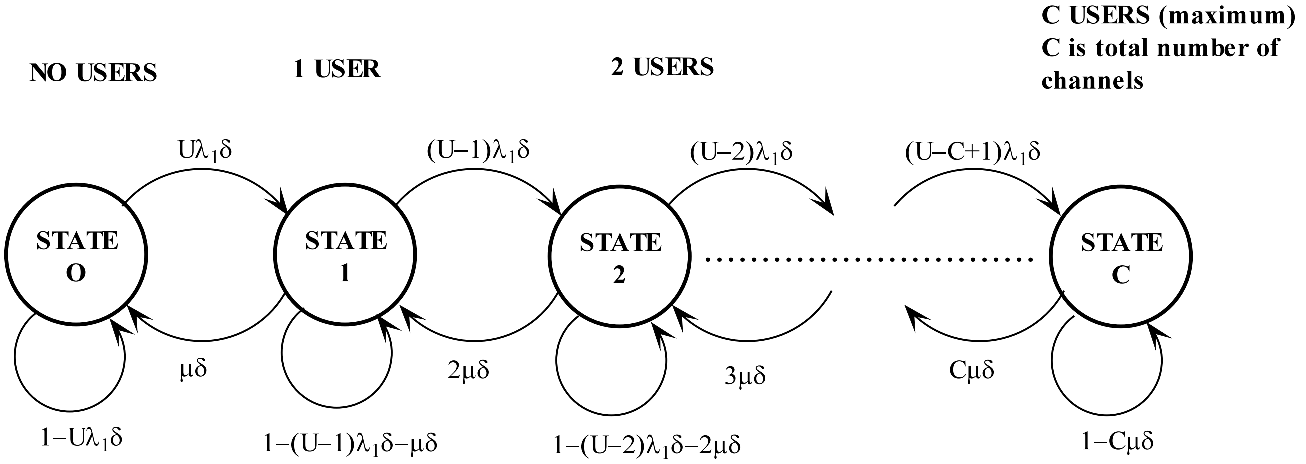
$\Pi_{i,i-1}$ are the same as in Erlang analysis,

$$\Pi_{i,i-1} = P\{ N_{k+1}=i-1 \mid N_k=i\} = P\{ \textit{one call out of i calls} \text{ finished in } [k\delta, (k+1)\delta]\}$$
$$= P\{s_1 < \delta\} + P\{s_2 < \delta\} + \ldots + P\{s_i < \delta\}$$
$$= i(1 - e^{-\mu\delta})$$
$$\approx i\mu\delta \quad \text{for } \mu\delta \ll 1 \text{, since the events are independent.}$$

Hence $\Pi_{i,i}$ becomes,

$$\Pi_{i,i} = 1-\Pi_{i,i+1}+\Pi_{i,i-1} = 1-(U-i)\lambda_1\delta - i\mu\delta \ .$$

The Markov chain for Engset model is



Applying the relation

$P\{N_k=n-1\}\ P\{N_{k+1}=n|\ N_k=n-1\}= P\{N_{k+1}=n\}\ P\{N_k=n-1\ |\ N_{k+1}=n\}$, i.e.

$P_{n-1}\Pi_{n-1,n} = P_n\Pi_{n,n-1}$,

We obtain

$$P_{n-1}[(U-n+1)\lambda_1\delta)=P_n\ (n\mu\delta)$$

$$\Rightarrow \qquad (U-n+1)\lambda_1 P_{n-1}= n\mu P_n \ .$$

Thus

$$P_1= U(\lambda_1/\mu)\ P_0$$

$$P_2= (U-1)(\lambda_1/2\mu)\ P_1= (1/2)U(U-1)(\lambda_1/\mu)^2\ P_0$$

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$$P_n= (U-n+1)(\lambda_1/n\mu)\ P_{n-1}= U(U-1)(U-2)...\ (U-n+1)(\lambda_1/\mu)^n\ P_0\ /n!$$

But,

$U(U-1)(U-2)\ldots(U-(n-1))/n! \quad = U!/[n!(U-n)!]$

$$= \binom{U}{n}$$

Therefore the probability of finding the system at state n is

$$P_n = \binom{U}{n}(\lambda_1/\mu)^n P_0 .$$

$\sum_{i=0}^{C} P_i = 1$ **yields,**

$$\sum_{i=0}^{C}\binom{U}{i}(\lambda_1/\mu)^i P_0 = 1 \qquad \Rightarrow P_0 = \frac{1}{\sum_{i=0}^{C}\binom{U}{i}\left(\frac{\lambda_1}{\mu}\right)^i}$$

Then

$$P_n = \frac{\binom{U}{n}\left(\frac{\lambda_1}{\mu}\right)^n}{\sum_{i=0}^{C}\binom{U}{i}\left(\frac{\lambda_1}{\mu}\right)^i}$$ **for any state, and**

$$P_C = \frac{\binom{U}{C}\left(\frac{\lambda_1}{\mu}\right)^C}{\sum_{i=0}^{C}\binom{U}{i}\left(\frac{\lambda_1}{\mu}\right)^i}$$ **, when all channels are busy.**

P_C is *time blocking probability*. Consider a long period T. The system spends the time $P_C T$ at state C. P_C is the proportion of time that the system spends at state C.

***Call blocking* is the probability of an arriving call finds the system at state C. Call blocking is different than P_C in this model, because the call arrivals are not a Poisson process, but it is state dependent.**

During a long period of time T, the total length of time that the system stays at state n is $P_n T$. Hence there are $(U-n)\lambda_1(P_n T)$ call arrivals during this period, which find the system at state n.

Now, the total number of calls arriving during T is

$$\sum_{i=0}^{C} (U-i)\lambda_1(P_iT) = T\lambda_1 \sum_{i=0}^{C} (U-i)P_i$$

$$= T\lambda_1 \sum_{i=0}^{C} (U-i)\binom{U}{i}(\lambda_1/\mu)^i P_0$$

$$= T\lambda_1 \sum_{i=0}^{C} U\binom{U-1}{i}(\lambda_1/\mu)^i P_0$$

since $(U-i)\binom{U}{i} = U\binom{U-1}{i}$.

Hence, the proportion of arriving calls, which find the system at state C, to the total call arrivals is

$$P_{BC} = \frac{(U-C)\lambda_1(P_C T)}{T\lambda_1 \sum_{i=0}^{C} U\binom{U-1}{i}\left(\frac{\lambda_1}{\mu}\right)^i P_0}$$

$$= \frac{(U-C)\binom{U}{C}\left(\frac{\lambda_1}{\mu}\right)^C P_0}{U\sum_{i=0}^{C}\binom{U-1}{i}\left(\frac{\lambda_1}{\mu}\right)^i P_0}$$

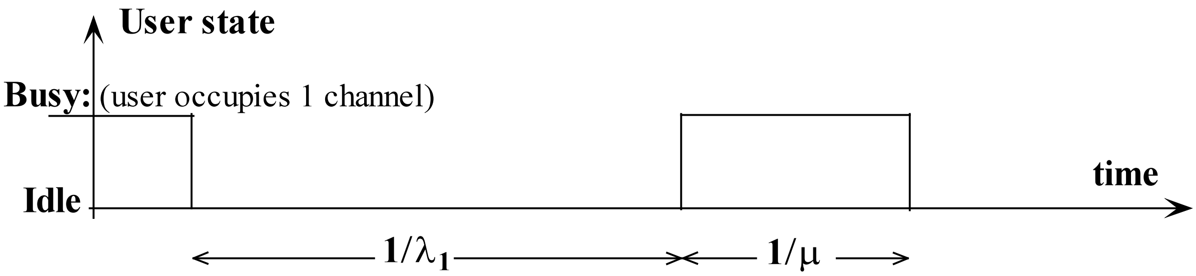
$$= \frac{\binom{U-1}{C}\left(\frac{\lambda_1}{\mu}\right)^C}{\sum_{i=0}^{C}\binom{U-1}{i}\left(\frac{\lambda_1}{\mu}\right)^i}.$$

λ in Erlang model is the call arrival rate for the entire user population. When U is not much larger than C, we must refine the call arrival rate definition.

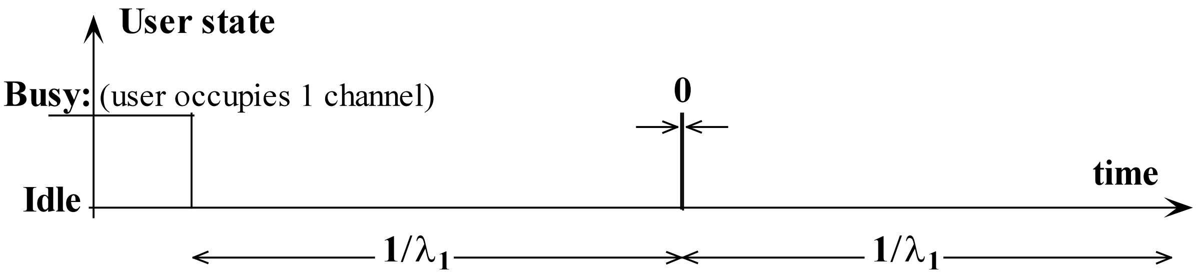
Each user makes a call attempt within $1/\lambda_1$ time units (sec, min or hr), on the average, *when he/she is not already speaking in the network*. If the user gets connected, there is an average holding time of $H=1/\mu$. If the user is blocked, another call attempt period of $1/\lambda_1$, on the average, starts. Hence

each user is either *idle* for $1/\lambda_1$ time units, or *busy* for $1/\mu$ time units *if he gets connected*.

Call is connected at the end of idle period



Call is blocked at the end of idle period



A user is blocked by a probability P_{BC}.
A user is connected by a probability $1-P_{BC}$.

Therefore one cycle is

$1/\mu + 1/\lambda_1$ **with probability** $1-P_{BC}$, **and**
$1/\lambda_1$ **with probability** P_{BC},

yielding an average cycle length of

$$(1/\mu + 1/\lambda_1)(1-P_{BC}) + (1/\lambda_1)(P_{BC})$$
$$= 1/\lambda_1 + (1/\mu)(1-P_{BC}).$$

Therefore the average traffic intensity offered by each user is

$$A_u = (1/\mu)/[1/\lambda_1 + (1/\mu)(1-P_{BC})]$$
$$= (\lambda_1/\mu)/[1+(\lambda_1/\mu)(1-P_{BC})] \text{ Erlang,}$$

i.e. user occupies a channel $1/\mu$ time units in $1/\lambda_1+(1/\mu)(1-P_{BC})$ time units. Note that A_u and A ($=UA_u$) now depends on μ and P_{BC} as well as on λ_1.

λ_1/μ can be written in terms of traffic intensity as,

$$\lambda_1/\mu = A_u/[1-(1-P_{BC})A_u]$$
$$= A/[U-(1-P_{BC})A].$$

$(1-P_{BC})A$ is, again, the carried traffic.